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A different approach for calculation of stress intensity factors in continuous fiber reinforced metal matrix composites

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Abstract

In this paper, a numerical model developed for the analysis of a cylindrical element of matrix containing a single fiber is presented. A ring-shaped crack is assumed at interface of fiber and matrix. Both layers in the model are bonded perfectly with the exception of the crack faces. Contact elements, which have bonded feature, are used between fiber and matrix. Displacement correlation method is used to calculate opening-mode and sliding-mode stress intensity factors. These results obtained from the analysis help to understand the debonding phenomenon between fiber and matrix interface. Effects of the mechanical properties of fiber and matrix on direction of crack propagation are also discussed.

Keywords: Stress intensity factor; Finite element analysis; Debonding; Fiber–matrix interface

1. Introduction

Adhesion between fiber and matrix is controlled by properties of the interface. Generally high degree of adhesion is desirable to provide efficient transfer of load between fiber and matrix. The strength of the adhesion controls properties of composite in a direction transverse to fibers as well as shear properties. Continuous fiber reinforced metal matrix composites are attractive as a material for high-performance components in aeroplane jet engines, for example, for compressor disks or blades (Masud et al., 1998; Peters et al., 2001; Rauchs et al., 2002). During service of highly loaded components fatigue cracks can be initiated. After initiation, these fatigue cracks can grow until final failure occurs.

Gdoutos et al. (1999) studied singular stress field in the neighborhood of the periphery of an annular crack. The case of fiber debonding originating from the annular crack was also considered. In the study, K_I and K_{II} stress intensity factors and energy release rates were calculated. Liu and Kagawa (2000) have derived the energy release rate for an interfacial debonding of a crack in a ceramic–matrix composite by the use of the Lame solution for an axisymmetric cylindrical fiber/matrix model. Aslantaş and Taşgetiren (2002) a numerical solution is carried out for the problem of interface crack. Variations in the stress

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intensity factors K_I and K_{II}, with load position are obtained for various cases such as different combinations of material of coating layer and substrate, changes in the coefficient of friction on the surface. Fatigue crack initiation in SiC fiber (SCS-6) reinforced titanium has been analysed on the basis of a finite element model by Xia et al. (2000). Their results showed that the formation of matrix crack largely depends on the applied stress and reaction layer thickness. Bjerken and Persson presented a method for obtaining the complex stress intensity factor (or alternatively the corresponding energy release rate and mode mixity) for an interface crack in a bimaterial using a minimum number of computations (Bjerken and Persson, 2001). Dirikolu and Aktas carried out a comparative study regarding the determination of stress intensity factors for nonstandard thin composite plates (Dirikolu and Aktaş, 2000). Carbon–epoxy composite plates were also considered for the study.

In generally, stress intensity factors at crack tip is calculated based on energy release rate or J integral (Rauchs et al., 2002; Gdoutos et al., 1999; Liu and Kagawa, 2000). In this study, a numerical analysis has been carried out for analysis of a cylindrical element of matrix with a single fiber with a ring-shaped crack at the interface. Special contact elements, which have bonded feature, are used between fiber and matrix. Displacement correlation method is used to calculate K_I and K_{II}.

2. Numerical analysis

2.1. Model description

Fig. 1 shows a micromechanical model used in this study. A ring-shaped crack is assumed at the interface between fiber and matrix. Both layers in the model are bonded perfectly with the exception of the crack faces. The sliding behaviour at the interface is characterised by Coulomb's friction law in which the coefficient of friction is assumed to be zero. Cylindrical model showing a fiber and matrix is given in Fig. 1.

2.2. Finite element model

Numerical modelling is done by the ANSYS 6.1 finite element program. Fig. 2 shows the configuration of a crack at the interface between fiber and matrix. Eight nodes isoparametric finite element elements are

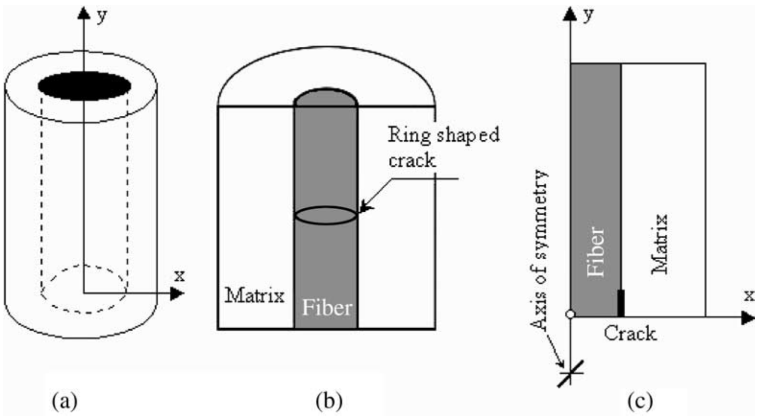

Fig. 1. (a) The composite cylinder model, (b) cross-section of continuous fiber reinforced composite, (c) axisymmetric model of the composite with crack.

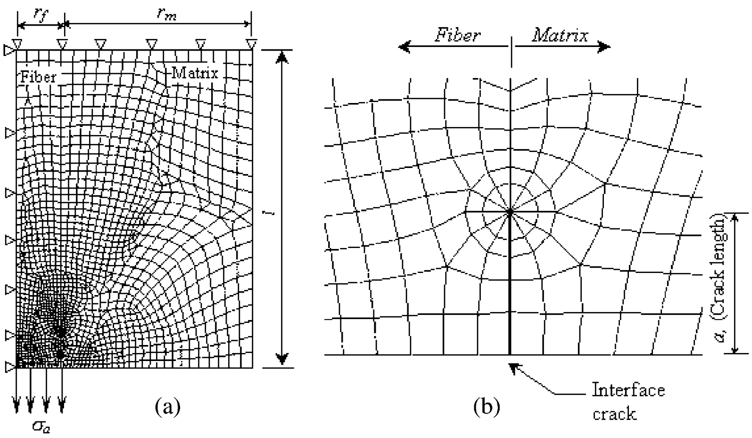

Fig. 2. (a) Finite element mesh and boundary conditions, (b) quarter point elements at the interface crack tip.

used for the model of the solution domain except those contacting the crack tip. These elements are six nodes triangular quarter point elements. The element and node numbers are 4512 and 6680 respectively.

The elements near the crack are taken as small as possible in order to simulate the stress distribution and deformation near the crack more accurately, as shown in Fig. 2b. TARGE169 and CONTA172 contact elements are used to represent various 2-D contacting surfaces between fiber and matrix. The target and associated contact surfaces are identified by a shared real constant set. This real constant set includes all real constants for both the target and contact elements. For the surfaces between fiber and matrix, except the crack surfaces, the property of bonded contact elements is used.

2.3. Calculation of stress intensity factors

The mixed-mode crack propagation scheme can be implemented by finite element or boundary element analysis. The advantage of such a numerical methods is that it calculates the stress intensity factors more accurately in terms of near crack tip nodal displacements (Chan et al., 1970) which is called displacement correlation method. In this analysis, displacement correlation method is used. The definitions are given in Fig. 3 for the application of the method.

After finite element or boundary element solutions for cracked structure is obtained, nodal displacement value of nodes a–e (Fig. 3a) is determined. Opening mode K_{I} and shear mode K_{II} are defined as (Tan and Gao, 1990),

$$K_{\mathrm{I}} = \sqrt{\frac{2\pi}{L}} D_1[v^{\mathrm{e}} - 4v^{\mathrm{d}} + 3v^{\mathrm{a}}] - D_2[v^{\mathrm{c}} - 4v^{\mathrm{b}} + 3v^{\mathrm{a}}] \tag{1}$$

$$K_{\mathrm{II}} = \sqrt{\frac{2\pi}{L}} D_1[u^{\mathrm{e}} - 4u^{\mathrm{d}} + 3u^{\mathrm{a}}] - D_2[u^{\mathrm{c}} - 4u^{\mathrm{b}} + 3u^{\mathrm{a}}] \tag{2}$$

where L is distance between nodes of a–c or a–e. v^i is the displacement along y at nodes a–e and u^i is displacement along x at nodes a–e. D_1 and D_2 can be expressed as:

$$D_1 = \frac{(1+\gamma)\lambda_0}{\cosh(\pi\varepsilon)} \cdot \frac{G_1}{\kappa_1 \mathrm{e}^{\pi\varepsilon} + \gamma \mathrm{e}^{-\pi\varepsilon}} \tag{3}$$

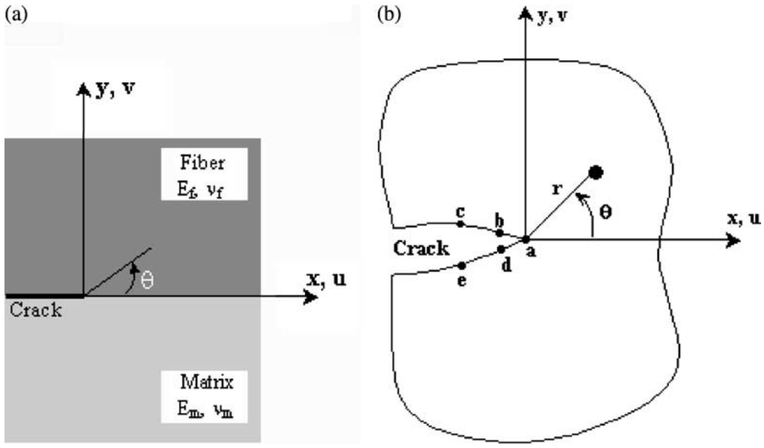

Fig. 3. (a) The interface crack between fiber and matrix, (b) nodes at the crack surfaces.

$$D_2 = \frac{(1+\gamma)\lambda_0}{\cosh(\pi\varepsilon)} \cdot \frac{G_2}{\kappa_2 \gamma \mathrm{e}^{-\pi\varepsilon} + \mathrm{e}^{\pi\varepsilon}} \tag{4}$$

$$\varepsilon = \frac{1}{2\pi} \ln\gamma, \quad \gamma = \frac{G_1 + \kappa_1 G_2}{G_2 + \kappa_2 G_1}, \quad \lambda_0 = \left(\frac{1}{4} + \varepsilon^2\right)^{1/2} \tag{5}$$

where κ is defined for axisymmetric problems $3 - 4v_i$. ε is the bimaterial constant, and G_1, G_2, v_1 and v_2 are shear modulus and Poisson's ratios for respective material. Shear modulus can be written as,

$$G_i = \frac{E_i}{2(1+v_i)} \tag{6}$$

When the mode I and II stress intensity factors are known, the predicted crack propagation angle can be estimated under mixed-mode loading. The most widely accepted method is maximum principal stress theory (Erdogan and Sih, 1963). According to this theory the crack propagates in a direction perpendicular to maximum tangential stress. In the present study the angle of crack growth is obtained from the maximum tangential stress theory.

$$\theta = 2\tan^{-1}\left[\frac{1}{4}\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}} \pm \sqrt{\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}}\right)^2 + 8}\right)\right] \tag{7}$$

3. Results and discussion

3.1. Stress and deformation at the crack tip

For the evaluation of stress intensity factors, K_{I} and K_{II} at the interfacial crack tip the relative displacements, Δy and Δx, of five points on upper and lower crack faces are determined by using the ANSYS 6.1 finite element software. In Fig. 4, Von-Misses stress distribution at the crack tip is given for $a/r_{\mathrm{f}} = 0.5$ and $E_{\mathrm{f}}/E_{\mathrm{m}} = 5$. Stress values are normalized with σ_{a}, applied stress on fiber. Fig. 5 displays a profile of deformed mesh for three different $E_{\mathrm{f}}/E_{\mathrm{m}}$ ratios. It is observed from Fig. 5 that elastic deformation at the

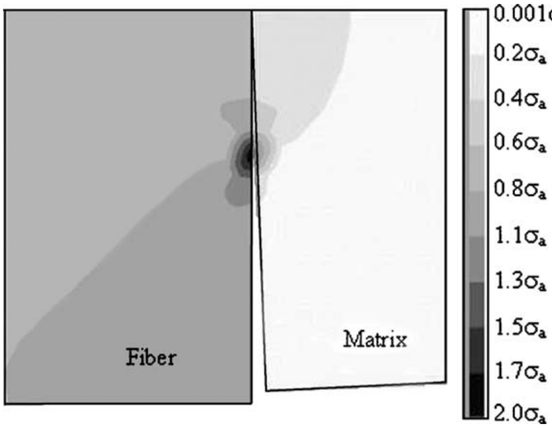

Fig. 4. Nondimensional Von-Misses stress distribution at the crack tip for $a/r_f = 0.5$ and $E_f/E_m = 5$.

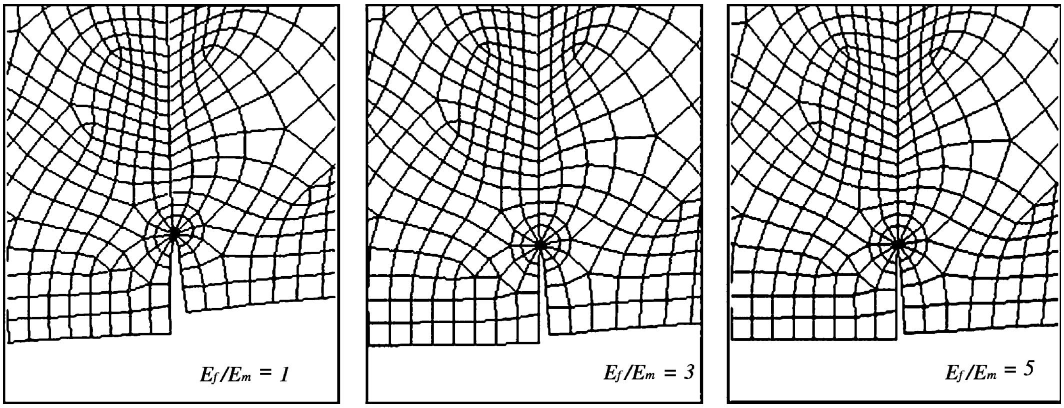

Fig. 5. Deformed mesh of the composite material for $a/r_f = 0.3$ (displacement multiplier 20×).

crack faces decreases as the E_f/E_m ratio increases. So stress intensity factors will decrease as node displacement values at the crack faces decreases.

3.2. Variation in stress intensity factors

Fig. 6 shows the variation of the normalized K_I stress intensity factor against the normalized interface crack. Fig. 7 also shows the variation of the K_{II}. Both K_I and K_{II} curves are obtained for four different E_f/E_m values. Opening mode K_I takes positive values for all E_f/E_m values and the applied stress on fiber exerts to open the crack faces. But, note that for all values of E_f/E_m, K_I decreases as the interface crack length increases. In addition to this, K_I decreases as the value of E_f/E_m increases (Fig. 6).

K_{II} affects the trajectory of the path of propagation of crack. Erdogan and Sih (1963) have shown that a crack continues to advance in its own plane when it is subjected to pure mode I. The presence of positive K_{II} deflects the crack in $-\theta$ direction, and negative K_{II} causes the crack to deviate in $+\theta$ direction according to Fig. 3. It can be seen from Figs. 6 and 7 that absolute K_{II} values are much higher than K_I values. Furthermore, sliding mode K_{II} stress intensity factor is affected more than opening mode K_I stress intensity

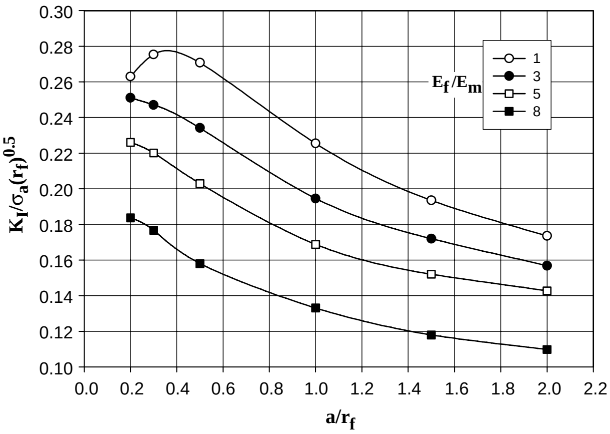

Fig. 6. Nondimensional K_I stress intensity factor versus interface crack length for different E_f/E_m values.

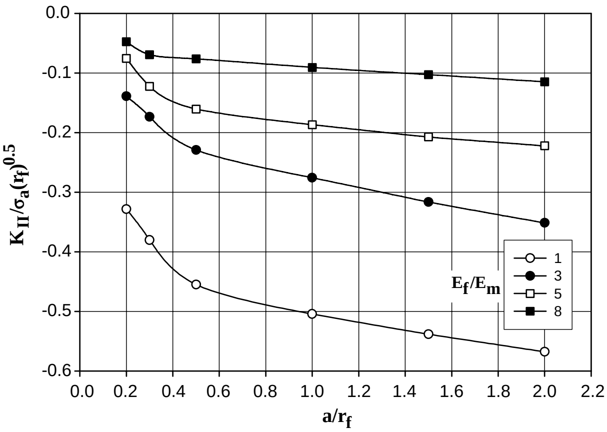

Fig. 7. Nondimensional K_{II} stress intensity factor versus interface crack length for different E_f/E_m values.

factor as the E_f/E_m value increases. These results indicate that deformation fields in the vicinity of the crack tip are dominated by shear mode. Namely, the debonding of x and y most probably occurs due to shear mode under axial loading.

In Fig. 8, kinking angle of the interface crack against crack length and different fiber matrix materials are given. It can be seen from the Fig. 8, the all crack kinking angles are positive due to negative shear mode K_{II} stress intensity factor.

4. Conclusions

The present study has provided a linear elastic fracture mechanics analysis of interface crack problem in continuous fiber reinforced metal matrix composites. The finite element mesh is formed for cylindrical element of matrix with a single fiber. A ring-shaped crack is assumed between fiber and matrix. Displacement correlation method is used to calculate K_I and K_{II} stress intensity factors at the crack tip.

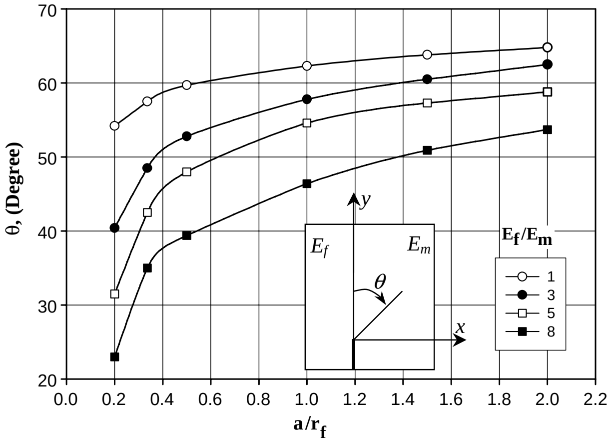

Fig. 8. Crack kinking angle versus nondimensional crack length and different E_f/E_m values.

The analysis results show that absolute K_{II} values are much higher than K_I values. Sliding mode K_{II} stress intensity factor is affected more than opening mode K_I stress intensity factor as the E_f/E_m value increases. The deformation fields in the vicinity of the crack tip are dominated by shear mode. Debonding between fiber and matrix most probably occurs due to shear mode under axial loading.

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